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Engineering Notes

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Residual Shaft Bending by Helicopter Rotors

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Nomenclature

e = offset of flapping hinge
$\hat{\boldsymbol{e}}_r$ = unit vector in direction of residual shaft bending moment
$\boldsymbol{f}\,dr$ = force on blade element
h = height of rotor hub above c.g.
J = blade moment of inertia in flapping
K = blade first moment of mass
M_r = residual shaft bending moment
N_b = number of blades
Q = instantaneous torque
Q_0 = value of Q, for untilted rotor
r = distance from hub measured along the blade
T = main rotor thrust
$\boldsymbol{x}$ = radius vector (x, y, z)
x = shaft system coordinate positive forward
y = shaft system coordinate positive to the right
z = shaft system coordinate positive down
β = blade flapping angle
$\delta\psi$ = angle of change in control direction caused by residual bending
ζ_r = ratio of residual bending to normal control moment
θ_{tp} = pitch inclination of the tip plane relative to the shaft
ϕ_{tp} = roll inclination of the tip plane relative to the shaft
Ψ = blade azimuth angle
Ω = shaft rotational rate

Superscript

$-$ = averaging over a revolution

I. Introduction

ATTITUDE control in helicopters is achieved by tilting the rotor tip plane relative to the shaft. This causes the thrust, which is roughly perpendicular to the tip plane, to exert a moment around the aircraft c.g. Additional control moments arise when blade flapping is restrained elastically, as with a rigid rotor or when the flapping hinges are offset. These hub moments are beneficial from the control point of view, but present a cost from the strength and fatigue perspective.

There is a third source of control moments that is usually overlooked. We call them residual bending moments. The residual moment is also introduced into the shaft, but unlike the normal hub moment, it is there even for a teetering rotor or an articulated rotor with no flap hinge offset.

The moment generated by the blades is a vector. The component parallel to the shaft is the torque, which the shaft transmits to the engine. The component parallel to the flapping hinge is the normal hub moment, which for a teetering rotor, must vanish. The third component, perpendicular to the other two, is the residual bending moment; it is reacted by the shaft in bending. The direction of the residual bending moment rotates with the shaft. Averaged over a revolution, residual bending can give rise to a net pitching or a rolling moment. For teetering and other similarly hinged rotors, residual bending is the only shaft bending that takes place. For other configurations, residual bending is superimposed on the shaft bending arising from the familiar hub moment.

Torsion of the blades, reacted by the pitch horns, is excluded from this discussion. This precludes any residual shaft bending so long as the blades are perpendicular to the shaft. However, residual bending moments occur when β does not vanish. In particular, when the tip plane is tilted relative to the shaft, residual bending manifests itself.

In Ref. 1, an inertial residual effect was explored that affects only multiblade rotors, is of third-order in the rotor tilt angle, and tends to reverse the normal control moments. The residual effect addressed here is aerodynamic, is first-order in the tip plane tilt, and is a cross-coupling effect: pitching of the tip plane gives rise to a residual roll moment and vice versa.

One can see this intuitively in Fig. 1, which shows a flat rotor disk pitched forward relative to the shaft. Each blade is subject to an aerodynamic force (a combination of section lift and drag[2]) that tends to retard its rotation. These forces act above the hub in the rear of the disk and below the hub in the front. In either case, a rolling moment to the left is produced.

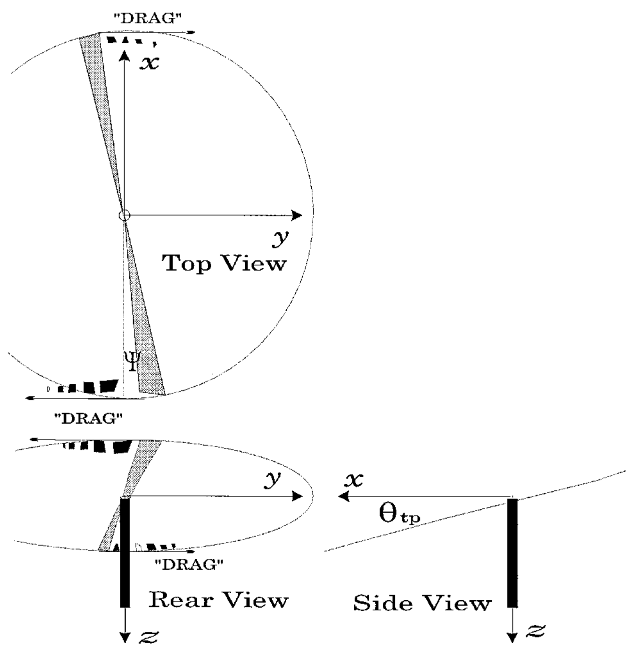

Fig. 1 Geometry of the inclined disk and the resulting moments.

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II. Analytic Derivation

We address a flat disk (Fig. 1). For simplicity, Ω is assumed to be constant. Reference 1 shows that, for a two-bladed rotor, this cannot be strictly true, but the effect of this assumption on the effect addressed here is negligible.

For a flat disk inclined by θ_{tp} (Fig. 1), the following purely geometrical relationships, developed in Ref. 1, apply: the flapping angle depends on azimuth through

$$\tan\beta = -\cos\Psi \tan\theta_{tp} \tag{1}$$

The trajectory of r is

$$x = -r\cos\Psi\cos\beta \tag{2}$$

$$y = r\sin\Psi\cos\beta \tag{3}$$

$$z = -r\sin\beta \tag{4}$$

The direction normal to the shaft and the flapping hinge is given by the unit vector

$$\hat{\boldsymbol{e}}_r = (-\cos\Psi, \sin\Psi, 0) \tag{5}$$

The moment induced by the blade element at point r is $\boldsymbol{x} \times \boldsymbol{f}$. The residual bending component is given by the triple product

$$\mathrm{d}\,M_r = \boldsymbol{x} \times \boldsymbol{f} \cdot \hat{\boldsymbol{e}}_r \tag{6}$$

Similarly, the contribution of the same blade element to the torque transmitted to the shaft is

$$\mathrm{d}Q = \boldsymbol{x} \times \boldsymbol{f} \cdot \hat{k} \tag{7}$$

Expressed in terms of the components of the vectors, the triple products become

$$\mathrm{d}M_r = \begin{vmatrix} x & f_x & -\cos\Psi \\ y & f_y & \sin\Psi \\ z & f_z & 0 \end{vmatrix} \tag{8}$$

$$\mathrm{d}Q = \begin{vmatrix} x & f_x & 0 \\ y & f_y & 0 \\ z & f_z & 1 \end{vmatrix} \tag{9}$$

Now develop both determinants by the last row. It is easy to check that the term containing f_z vanishes in both cases. This is to be anticipated, since a vertical force creates a flapping moment only. This leaves, after using Eqs. (2–4)

$$\mathrm{d}M_r = -r\sin\beta(f_x\sin\Psi + f_y\cos\Psi) \tag{10}$$

$$\mathrm{d}Q = -r\cos\beta(f_x\sin\Psi + f_y\cos\Psi) \tag{11}$$

Putting Eqs. (10) and (11) together and invoking Eq. (1), one has

$$\mathrm{d}M_r = \tan\beta\,\mathrm{d}Q = -\cos\Psi\tan\theta_{tp}\,\mathrm{d}Q \tag{12}$$

This is readily integrated over the whole blade and summed over the blades to yield the relationship

$$M_r = \tan\beta\,Q = -\cos\Psi\tan\theta_{tp}\,Q \tag{13}$$

between the instantaneous values of M_r and Q.

We next average over a revolution. This is done over the azimuth angle (it is here that the assumption of constant Ω is invoked). The average pitching moment is

$$\bar{M}_{ry} = \frac{1}{2\pi}\int_0^{2\pi} M_r\sin\Psi\,\mathrm{d}\Psi = -\tan\theta_{tp}\frac{1}{2\pi}\int_0^{2\pi} Q\sin\Psi\cos\Psi\,\mathrm{d}\Psi \tag{14}$$

and the average rolling moment

$$\bar{M}_{rz} = \frac{1}{2\pi}\int_0^{2\pi} M_r(-\cos\Psi)\,\mathrm{d}\Psi = \tan\theta_{tp}\frac{1}{2\pi}\int_0^{2\pi} Q\cos^2\Psi\,\mathrm{d}\Psi \tag{15}$$

Residual bending vanishes when the rotor inclination θ_{tp} does, and is first order in θ_{tp}. First-order results may be obtained by using Q_0, namely Q for the untilted rotor in Eqs. (14) and (15). With this done, the quadratures in Eqs. (14) and (15) can be carried out explicitly in the case of idealized hover or axial flight. By idealized we mean that rotor thrust is vertical and all asymmetries related to tail rotor thrust are neglected. In this case, $Q = Q_0 = \bar{Q}$ is independent of Ψ. To first-order in θ_{tp}, Eqs. (14) and (15) reduce to

$$\bar{M}_x = \tfrac{1}{2}\theta_{tp}\bar{Q} \tag{16}$$

$$\bar{M}_y = 0 \tag{17}$$

Under these conditions, and to first-order, residual bending is a pure cross-coupling effect.

In idealized hover, all horizontal directions are equivalent. If a roll inclination to the shaft were introduced, in addition to the pitch inclination, then, by symmetry, Eq. (17) would be transformed into

$$\bar{M}_y = -\tfrac{1}{2}\phi_{tp}\bar{Q} \tag{18}$$

The last three equations are stated in terms of $\bar{Q}$ rather than Q_0 for the purpose of numerical studies beyond the simplifying conditions assumed in the derivation.

III. Computer Simulation

Figure 2 is a plot of residual bending against rotor inclination. The data are for the Robinson R22. The prediction of Eq. (16) is superimposed on a curve obtained by computer simulation. The simulation code that was used is Bladehelo, a rotor model developed at the University of Alabama.[1,3] The code is based on the rigorous mechanics of individual flapping blades and does not invoke any small angle assumptions. It is seen

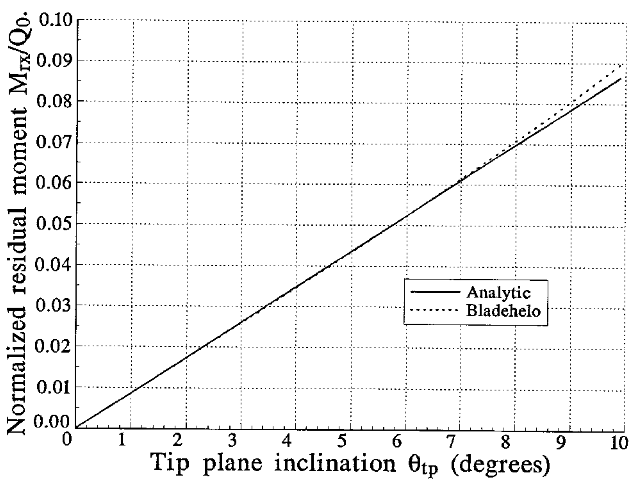

Fig. 2 Normalized residual shaft bending vs. disk inclination.

Table 1 Residual bending and tip plane inclination in forward flight[a]

Speed, kn	θ_{tp}, deg	$\frac{1}{2}\theta_{tp}$, rad	M_x/Q
0	−4.9692	−0.04336	−0.04327
10	−4.5464	−0.03967	−0.03963
20	−4.0913	−0.03570	−0.03574
30	−3.5879	−0.03131	−0.03142
40	−3.0461	−0.02658	−0.02670
50	−2.4846	−0.02168	−0.02181
60	−1.9201	−0.01676	−0.01693
70	−1.3755	−0.01200	−0.01219
80	−0.8511	−0.00743	−0.00761
90	−0.3485	−0.00304	−0.00311
100	0.1377	0.00120	0.00139

[a]Fixed shaft, cyclic input is 5-deg forward.

that, within the range of inclinations allowed in the Robinson R22, Eq. (16) is a very close approximation.

The data plotted in Fig. 2 are for idealized hover. The data were obtained by fixing the shaft, adjusting the collective to eliminate vertical acceleration, and then applying the longitudinal cyclic input. The tip plane followed the swashplate to within 0.1 deg. The residual bending moment is shown nondimensionalized by Q_0.

To first-order, Eq. (16) could be replaced with $\bar{M}_{ry} = \frac{1}{2}\theta_{tp}Q_0$. What is plotted in Fig. 2 is $\bar{M}_{ry}/Q_0$. Equation (16) was left in terms $\bar{Q}$, because it turns out that agreement with the computer simulation is even closer that way. Had the Bladehelo results been plotted as $\bar{M}_{ry}/\bar{Q}$, the two curves in Fig. 2 would then be indistinguishable.

Table 1 shows that Eq. (16) remains very nearly true in forward flight. The data in Table 1 were computed with the shaft fixed vertical and with a constant forward cyclic input of 5 deg. θ_{tp} varies with speed because of blowback. Table 1 presents the values of θ_{tp} that resulted and compares $\frac{1}{2}\theta_{tp}$ to $M_x/\bar{Q}$.

The inputs into Bladehelo were selected to represent a Robinson R22 β. An assessment of the accuracy of the numerical computation in Appendix B of Ref. 1 shows that the moments produced by Bladehelo are accurate to a fraction of a percent. The data presented show that Eqs. (16) and (18) remain very nearly true over a wide range of flight conditions.

IV. Magnitude and Significance of Residual Bending

In first-order, residual bending, like the normal control moment, is proportional to the tip plane inclination. For teetering rotors, the ratio of the residual bending moment [Eq. (16)] to the control moment that results from tilting the thrust is

$$\zeta_{r,\text{teeter}} = \bar{Q}/2Th \tag{19}$$

When this is applied to the Robinson R22 in out of ground effect (OGE) hover at full takeoff (TO) power (131 hp), residual bending amounts to 7.9% of the normal control moment. This is reduced to 7.5% at the maximum continuous power (124 hp), and is further reduced in proportion to power required under more favorable conditions. (The numbers are based on the assumption that 90% of the engine power reaches the main rotor.)

In the case of offset flapping hinges, Eq. (19) must be replaced by

$$\zeta_r = \frac{\bar{Q}}{2Th + N_b eK\Omega^2} \tag{20}$$

to account for conventional hub moments. Applied to the AH-64, Eq. (20) computes as 4%.

The effect of residual bending is to make the total effective control moment act in a direction that is off (in a conventional American helicopter, to the left) from the direction in which the tip plane is tilted. The difference is

$$\delta\psi_r = \arctan\zeta_r \tag{21}$$

which amounts to 4 deg in the Robinson R22 and 2 deg in the AH-64.

The residual cross-coupling effect may already be embedded in those codes that, like Bladehelo, are based on full and correct blade dynamics. However, it is unfamiliar to the community, as an effect to be anticipated and included in preliminary estimates.

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Multiobjective Genetic Algorithm for Multidisciplinary Design of Transonic Wing Planform

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Introduction

FORMULATION of multidisciplinary optimization (MDO) presents organizational challenges for coupling analysis codes from each discipline.[1] A simple sequential optimization that executes each disciplinary optimization task in sequence cannot take advantage of the benefits from cross-disciplinary tradeoffs. Therefore, MDO requires multiobjective, system-level optimization.

Multiobjective optimization seeks to optimize the components of a vector-valued objective function. Unlike single objective optimization, the solution to this problem is not a single point, but a family of points known as the Pareto-optimal set. Each point in this set is optimal in the sense that no improvement can be achieved in one objective component that does not lead to degradation in at least one of the remaining components. Conventional optimization techniques seek such solutions one-by-one. Genetic algorithms (GAs), however, can search for many Pareto solutions in parallel by maintaining a population of solutions.[2,3] Furthermore, a number of Pareto solutions create a locus in the design space where tradeoffs

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